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Note

A Comment on the Paper "The Calculation of Large Reynolds Number Flow Using Discrete Vortices with Random Walk" by F. Milinazzo and P. G. Saffman

In a recent paper [1], Milinazzo and Saffman discussed random vortex methods for two-dimensional flow, including some of my work [2, 3]. Their conclusion is that uncritical use of these methods may lead to significant error. This conclusion is of course valid for any method, and the need for caution is compounded with vortex methods by the absence of precise error bounds and by practical ambiguities. However, the particular grounds for criticism chosen by Milinazzo and Saffman are inappropriate.

1. Milinazzo and Saffman present a history of these methods, starting with a private communication from Professor Moore and some unsuccessful tests by Professor Saffman. As should be clear from the text, Professors Moore and Saffman did not make the method work, did not pursue it, and did not publish it. Nor did they communicate it to me. Thus, the statements "Subsequently, the method was presented by Chorin ...," "Chorin replaced ...," etc., although conceivably true in some literal sense, create an unwarranted impression of collaboration and joint work.

2. If one considers the bottom of the Milinazzo–Saffman Fig. 1, where they plotted the error (i.e., the difference between the exact solution and the computed solution), and if one remembers that the Reynolds number is relatively high and that the calculation was carried out over three vortex rotation times, one sees that the method has worked well even in the hands of Milinazzo and Saffman, and even with their cutoff (see point 3, below). In particular there is no trace of numerical dissipation. The relative error (error divided by a norm of the solution) is even smaller, since $1 + 4\nu t \geqslant 1$. In later figures and graphs and in the discussion the error looks much larger because it has been divided by $4\nu t$, where $\nu = 1$ and t varies between 0 and $\pi/100$; i.e., the error has been arbitrarily multiplied by a factor which varies between infinity and about 10. In particular, this division leads to an infinite "error" when $t = 0$, i.e., before the calculation has even started, and when $\nu = 0$, when there is no error at all. The Saffman–Milinazzo conclusion that the "error" is "large" rests entirely on their unusual interpretation of the word "error."

If there is a point in the Saffman–Milinazzo discussion, it is the following: If one tries to resolve an extremely small variation in the solution by an approximate method, one may have to resort to heroic means. This is of course true with all approximate methods, and casts no particular doubt on their validity. This point is relevant here only if one believes the effects of viscosity to be small always.

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3. It should be obvious that the cutoff used by Milinazzo and Saffman, which is 1/50th of the average distance between vortices, is equivalent to no cutoff at all. As explained in [3], a reasonable cutoff is needed for the success of the method. In the Milinazzo–Saffman problem this may not matter because the nonlinear terms in their solution vanish identically, and all that is required of the cutoff is that it preserve the symmetries which allow the cancellations.

References

1. F. Milinazzo and P. G. Saffman, *J. Computational Physics* **23** (1977), 300.
2. A. J. Chorin, *in* "Proceedings of the Third International Conference on Numerical Methods in Fluid Dynamics," Springer, New York, 1972.
3. A. J. Chorin, *J. Fluid Mech.* **57** (1973), 785.

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Alexandre J. Chorin

Department of Mathematics
University of California
Berkeley, California 94720